INCOMING 8TH GRADE

SUGGESTED SUMMER PRACTICE FOR

MATH

(YOU SHOULD ALSO SPEND 10-15 MINUTES PER DAY ON iReady)

Hello Future 8th Graders!

I'm looking forward to seeing you in the fall, and helping you mold your mind into a fine-tuned math machine. Really, not kidding.

In case you have some down time between fishing, swimming, camping, hiking, biking, and wakeboarding, here is some suggested math practice to keep and get you tuned up for jumping 8th grade math in August.

This is, of course, optional. You know your skills better than I do, and if you feel like, even after a year of Michele Cunningham's excellent tutelage, you could use an extra push to get good at this stuff, I'd highly recommend it!

Enjoy the summer,

Mr. Sarkisian

SIMPLIFYING FRACTIONS

The Giant One is also useful when simplifying or reducing fractions to lowest terms. Use the greatest common factor of the numerator and denominator for the Giant One. Divide the numerator and denominator by the greatest common factor and write the resulting fraction as a product with the Giant One. What remains is the simplified version of the fraction.

Example 1

Simplify $\frac{20}{24}$.

The greatest common factor of 20 and 24 is 4.

$$\frac{20}{24} = \frac{5}{6} \cdot \frac{4}{4} = \frac{5}{6} \cdot 1 = \frac{5}{6}$$

Example 2

Simplify $\frac{45}{60}$.

The greatest common factor of 45 and 60 is 15.

$$\frac{45}{60} = \frac{3}{4} \cdot \frac{15}{15} = \frac{3}{4} \cdot 1 = \frac{3}{4}$$

Problems

Simplify each fraction to lowest terms.

1. $\frac{10}{15}$	2. $\frac{9}{12}$	3. $\frac{20}{25}$	4. $\frac{14}{49}$	5. $\frac{24}{48}$
6. $\frac{15}{27}$	7. $\frac{24}{32}$	8. $\frac{28}{49}$	9. $\frac{50}{54}$	10. $\frac{24}{36}$
11. $\frac{125}{100}$	12. $\frac{16}{64}$	13. $\frac{12}{28}$	14. $\frac{56}{96}$	15. $\frac{108}{81}$

Answers

1. $\frac{2}{3}$	2. $\frac{3}{4}$	3. $\frac{4}{5}$	4. $\frac{2}{7}$	5. $\frac{1}{2}$
6. $\frac{5}{9}$	7. $\frac{3}{4}$	8. $\frac{4}{7}$	9. $\frac{25}{27}$	10. $\frac{2}{3}$
11. $\frac{5}{4}$	12. $\frac{1}{4}$	13. $\frac{3}{7}$	14. $\frac{7}{12}$	15. $\frac{4}{3}$

EQUIVALENT FRACTIONS

Fractions that name the same value are called equivalent fractions, such as $\frac{2}{3}=\frac{6}{9}$. One method for finding equivalent fractions is to use the Multiplicative Identity (Identity Property of Multiplication), that is, multiplying the given fraction by a form of the number 1 such as $\frac{2}{2}$, $\frac{5}{5}$, etc. In this course we call these fractions a "Giant One." Multiplying by 1 does not change the value of a number.

For additional information, see the Math Notes boxes in Lesson 3.1.1 of the *Core Connections, Course 1* text or Lesson 1.2.8 of the *Core Connections, Course 2* text.

Example 1

Find three equivalent fractions for $\frac{1}{2}$.

$\frac{1}{2}\cdot\boxed{\frac{2}{2}}=\frac{2}{4}$ $\qquad$ $\frac{1}{2}\cdot\boxed{\frac{3}{3}}=\frac{3}{6}$ $\qquad$ $\frac{1}{2}\cdot\boxed{\frac{4}{4}}=\frac{4}{8}$

Example 2

Use the Giant One to find an equivalent fraction to $\frac{7}{12}$ using 96ths: $\frac{7}{12}\cdot\boxed{1}=\frac{?}{96}$

Which Giant One do you use?

Since $\frac{96}{12}=8$, the Giant One is $\frac{8}{8}$: $\qquad$ $\frac{7}{12}\cdot\boxed{\frac{8}{8}}=\frac{56}{96}$

Problems

Use the Giant One to find the specified equivalent fraction. Your answer should include the Giant One you use and the equivalent numerator.

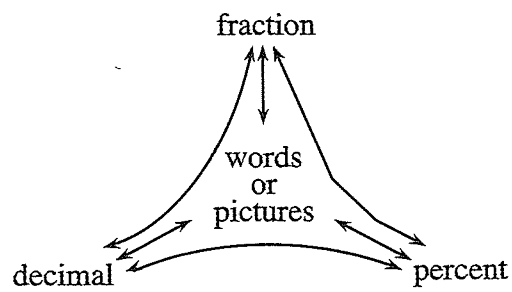
1. $\frac{4}{3}\cdot\boxed{1}=\frac{?}{15}$
2. $\frac{5}{9}\cdot\boxed{1}=\frac{?}{36}$
3. $\frac{9}{2}\cdot\boxed{1}=\frac{?}{38}$
4. $\frac{3}{7}\cdot\boxed{1}=\frac{?}{28}$
5. $\frac{5}{3}\cdot\boxed{1}=\frac{?}{18}$
6. $\frac{6}{5}\cdot\boxed{1}=\frac{?}{15}$

Answers

1. $\frac{5}{5}$, 20 $\quad$ 2. $\frac{4}{4}$, 20 $\quad$ 3. $\frac{19}{19}$, 171 $\quad$ 4. $\frac{4}{4}$, 12 $\quad$ 5. $\frac{6}{6}$, 30 $\quad$ 6. $\frac{3}{3}$, 18

FRACTION-DECIMAL-PERCENT EQUIVALENTS

Fractions, decimals, and percents are different ways to represent the same portion or number.



Representations of a Portion

For additional information, see the Math Notes boxes in Lessons 3.1.4 and 3.1.5 of the *Core Connections, Course 1* text, Lesson 2.1.2 of the *Core Connections, Course 2* text, or Lesson 1.1.1 of the *Core Connections, Course 3* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 5 materials or the *Core Connections, Course 2* Checkpoint 2 materials.

Examples

Decimal to percent:

Multiply the decimal by 100.

$(0.81)(100) = 81\%$

Fraction to percent:

Write a proportion to find an equivalent fraction using 100 as the denominator. The numerator is the percent.

$\frac{4}{5} = \frac{x}{100}$ so $\frac{4}{5} = \frac{80}{100} = 80\%$

Decimal to fraction:

Use the digits in the decimal as the numerator.
Use the decimal place value name as the denominator. Simplify as needed.

a. $0.2 = \frac{2}{10} = \frac{1}{5}$ b. $0.17 = \frac{17}{100}$

Percent to decimal:

Divide the percent by 100.

$43\% \div 100 = 0.43$

Percent to fraction:

Use 100 as the denominator. Use the percent as the numerator. Simplify as needed.

$22\% = \frac{22}{100} = \frac{11}{50}$

$56\% = \frac{56}{100} = \frac{14}{25}$

Fraction to decimal:

Divide the numerator by the denominator.

$\frac{3}{8} = 3 \div 8 = 0.375$ $\frac{5}{8} = 5 \div 8 = 0.625$

$\frac{3}{11} = 3 \div 11 = 0.2727\ldots = 0.\overline{27}$

To see the process for converting repeating decimals to fractions, see problem 2-22 in the *Core Connections, Course 2* text, problem 9-105 in the *Core Connections, Course 3* text, or the Math Notes boxes referenced above.

Problems

Convert the fraction, decimal, or percent as indicated.

1. Change $\frac{1}{4}$ to a decimal.
2. Change 50% into a fraction in lowest terms.
3. Change 0.75 to a fraction in lowest terms.
4. Change 75% to a decimal.
5. Change 0.38 to a percent.
6. Change $\frac{1}{5}$ to a percent.
7. Change 0.3 to a fraction.
8. Change $\frac{1}{8}$ to a decimal.
9. Change $\frac{1}{3}$ to a decimal.
10. Change 0.08 to a percent.
11. Change 87% to a decimal.
12. Change $\frac{3}{5}$ to a percent.
13. Change 0.4 to a fraction in lowest terms.
14. Change 65% to a fraction in lowest terms.
15. Change $\frac{1}{9}$ to a decimal.
16. Change 125% to a fraction in lowest terms.
17. Change $\frac{8}{5}$ to a decimal.
18. Change 3.25 to a percent.
19. Change $\frac{1}{16}$ to a decimal. Change the decimal to a percent.
20. Change $\frac{1}{7}$ to a decimal.
21. Change 43% to a fraction. Change the fraction to a decimal.
22. Change 0.375 to a percent. Change the percent to a fraction.
23. Change $\frac{7}{8}$ to a decimal. Change the decimal to a percent.
24. Change $0.\overline{12}$ to a fraction
25. Change $0.\overline{175}$ to a fraction

Answers

1. 0.25	2. $\frac{1}{2}$	3. $\frac{3}{4}$	4. 0.75
5. 38%	6. 20%	7. $\frac{3}{10}$	8. 0.125
9. $0.3\overline{3}$	10. 8%	11. 0.87	12. 60%
13. $\frac{2}{5}$	14. $\frac{13}{20}$	15. $0.1\overline{1}$	16. $\frac{5}{4}$ or $1\frac{1}{4}$
17. 1.6	18. 325%	19. 0.0625; 6.25%	20. $0.\overline{142859}$
21. $\frac{43}{100}$; 0.43	22. $37\frac{1}{2}$%; $\frac{3}{8}$	23. 0.875; 87.5%	
24. $\frac{12}{99}=\frac{4}{33}$	25. $\frac{175}{999}$		

OPERATIONS WITH FRACTIONS

ADDITION AND SUBTRACTION OF FRACTIONS

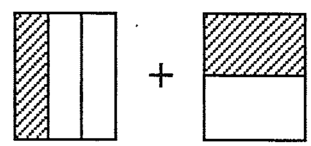
Before fractions can be added or subtracted, the fractions must have the same denominator, that is, a common denominator. We will present two methods for adding or subtracting fractions.

AREA MODEL METHOD

Step 1: Copy the problem.

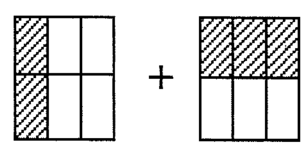
$\frac{1}{3} + \frac{1}{2}$

Step 2: Draw and divide equal-sized rectangles for each fraction. One rectangle is cut vertically into an equal number of pieces based on the first denominator (bottom). The other is cut horizontally, using the second denominator. The number of shaded pieces in each rectangle is based on the numerator (top). Label each rectangle, with the fraction it represents.



$\frac{1}{3} \quad + \quad \frac{1}{2}$

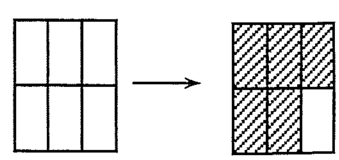
Step 3: Superimpose the lines from each rectangle onto the other rectangle, as if one rectangle is placed on top of the other one.



$\frac{2}{6} \quad + \quad \frac{3}{6}$

Step 4: Rename the fractions as sixths, because the new rectangles are divided into six equal parts. Change the numerators to match the number of sixths in each figure.

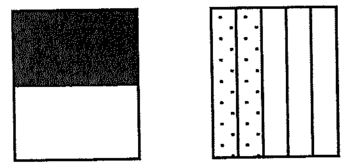
Step 5: Draw an empty rectangle with sixths, then combine all sixths by shading the same number of sixths in the new rectangle as the total that were shaded in both rectangles from the previous step.



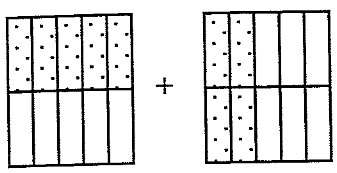
$\frac{5}{6}$

Example 1

$\frac{1}{2} + \frac{2}{5}$ can be modeled as:

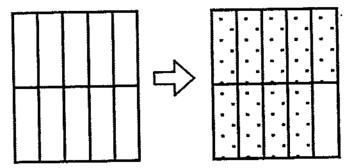


$\frac{1}{2}$ $\frac{2}{5}$



$\frac{5}{10}$ + $\frac{4}{10}$ $\quad \frac{5}{10} + \frac{4}{10}$

so

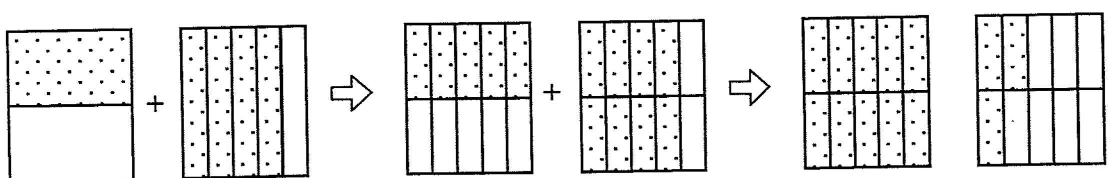


$\frac{9}{10}$

Thus, $\frac{1}{2} + \frac{2}{5} = \frac{9}{10}$.

Example 2

$\frac{1}{2} + \frac{4}{5}$ would be:



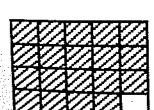
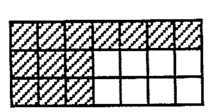
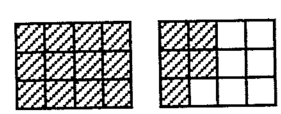
$\frac{1}{2}$ + $\frac{4}{5}$ ⇨ $\frac{5}{10}$ + $\frac{8}{10}$ ⇨ $\frac{13}{10} = 1\frac{3}{10}$

Problems

Use the area model method to add the following fractions.

1. $\frac{3}{4} + \frac{1}{5}$
2. $\frac{1}{3} + \frac{2}{7}$
3. $\frac{2}{3} + \frac{3}{4}$

Answers

1. $\frac{19}{20}$ 
2. $\frac{13}{21}$ 
3. $\frac{17}{12} = 1\frac{5}{12}$ 

IDENTITY PROPERTY OF MULTIPLICATION (GIANT ONE) METHOD

The Giant One, known in mathematics as the Identity Property of Multiplication or Multiplicative Identity, uses a fraction with the same numerator and denominator ($\frac{3}{3}$, for example) to write an equivalent fraction that helps to create common denominators. For additional information, see the Math Notes box in Lesson 3.1.2 of the *Core Connections, Course 1* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 3 materials.

Example

Add $\frac{1}{3} + \frac{1}{4}$ using the Giant One.

Step 1: Multiply both $\frac{1}{3}$ and $\frac{1}{4}$ by Giant **1**s to get a common denominator.

$$\frac{1}{3} \cdot \frac{4}{4} + \frac{1}{4} \cdot \frac{3}{3} = \frac{4}{12} + \frac{3}{12}$$

Step 2: Add the numerators of both fractions to get the answer.

$$\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

Problems

Find each sum or difference. Use the method of your choice.

1. $\frac{1}{3} + \frac{2}{5}$	2. $\frac{1}{6} + \frac{2}{3}$	3. $\frac{3}{8} + \frac{2}{5}$	4. $\frac{1}{4} + \frac{3}{7}$
5. $\frac{2}{9} + \frac{3}{4}$	6. $\frac{5}{12} + \frac{1}{3}$	7. $\frac{4}{5} - \frac{1}{3}$	8. $\frac{3}{4} - \frac{1}{5}$
9. $\frac{7}{9} - \frac{2}{3}$	10. $\frac{3}{4} + \frac{1}{3}$	11. $\frac{5}{6} + \frac{2}{3}$	12. $\frac{7}{8} + \frac{1}{4}$
13. $\frac{6}{7} - \frac{2}{3}$	14. $\frac{1}{4} - \frac{1}{3}$	15. $\frac{3}{5} + \frac{3}{4}$	16. $\frac{5}{7} - \frac{3}{4}$
17. $\frac{1}{3} - \frac{3}{4}$	18. $\frac{2}{5} + \frac{9}{15}$	19. $\frac{3}{5} - \frac{2}{3}$	20. $\frac{5}{6} - \frac{11}{12}$

Answers

1. $\frac{11}{15}$	2. $\frac{5}{6}$	3. $\frac{31}{40}$	4. $\frac{19}{28}$	5. $\frac{35}{36}$
6. $\frac{3}{4}$	7. $\frac{7}{15}$	8. $\frac{11}{20}$	9. $\frac{1}{9}$	10. $\frac{13}{12}=1\frac{1}{12}$
11. $\frac{3}{2}=1\frac{1}{2}$	12. $\frac{9}{8}=1\frac{1}{8}$	13. $\frac{4}{21}$	14. $-\frac{1}{12}$	15. $\frac{27}{20}=1\frac{7}{20}$
16. $-\frac{1}{28}$	17. $-\frac{5}{12}$	18. 1	19. $-\frac{1}{15}$	20. $-\frac{1}{12}$

To summarize addition and subtraction of fractions:

1. Rename each fraction with equivalents that have a common denominator.
2. Add or subtract only the numerators, keeping the common denominator.
3. Simplify if possible.

ADDITION AND SUBTRACTION OF MIXED NUMBERS

To subtract mixed numbers, change the mixed numbers into fractions greater than one, find a common denominator, then subtract. Other strategies are also possible. For additional information, see the Math Notes box in Lesson 4.1.3 of the *Core Connections, Course 1* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 4 materials.

Example

Find the difference: $3\frac{1}{5} - 1\frac{2}{3}$

$$3\tfrac{1}{5} = \tfrac{16}{5} \cdot \tfrac{3}{3} = \tfrac{48}{15}$$

$$-1\tfrac{2}{3} = \tfrac{5}{3} \cdot \tfrac{5}{5} = -\tfrac{25}{15}$$

$$= \tfrac{23}{15} = 1\tfrac{8}{15}$$

Problems

Find each difference.

1. $2\frac{1}{2} - 1\frac{3}{4}$
2. $4\frac{1}{3} - 3\frac{5}{6}$
3. $1\frac{1}{6} - \frac{3}{4}$
4. $5\frac{2}{5} - 3\frac{2}{3}$
5. $7 - 1\frac{2}{3}$
6. $5\frac{3}{8} - 2\frac{2}{3}$

Answers

1. $\frac{5}{2} - \frac{7}{4} \Rightarrow \frac{10}{4} - \frac{7}{4} \Rightarrow \frac{3}{4}$
2. $\frac{13}{3} - \frac{23}{6} \Rightarrow \frac{26}{6} - \frac{23}{6} \Rightarrow \frac{3}{6}$ or $\frac{1}{2}$
3. $\frac{7}{6} - \frac{3}{4} \Rightarrow \frac{14}{12} - \frac{9}{12} \Rightarrow \frac{5}{12}$
4. $\frac{27}{5} - \frac{11}{3} \Rightarrow \frac{81}{15} - \frac{55}{15} \Rightarrow \frac{26}{15}$ or $1\frac{11}{15}$
5. $\frac{7}{1} - \frac{5}{3} \Rightarrow \frac{21}{3} - \frac{5}{3} \Rightarrow \frac{16}{3}$ or $5\frac{1}{3}$
6. $\frac{43}{8} - \frac{8}{3} \Rightarrow \frac{129}{24} - \frac{64}{24} \Rightarrow \frac{65}{24}$ or $2\frac{17}{24}$

To add mixed numbers, it is possible to change the mixed numbers into fractions greater than one, find a common denominator, then add. Many times it is more efficient to add the whole numbers, add the fractions after getting a common denominator, and then simplify the answer. For additional information, see the Math Notes box in Lesson 4.1.3 of the *Core Connections, Course 1* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 4 materials.

Example

Find the sum: $8\frac{3}{4} + 4\frac{2}{5}$

$$8\frac{3}{4} = 8 + \frac{3}{4} \cdot \boxed{\frac{5}{5}} = 8\frac{15}{20}$$
$$+4\frac{2}{5} = 4 + \frac{2}{5} \cdot \boxed{\frac{4}{4}} = +4\frac{8}{20}$$
$$12\frac{23}{20} = 13\frac{3}{20}$$

Problems

Find each sum.

1. $5\frac{3}{4} + 3\frac{1}{6}$
2. $5\frac{2}{3} + 8\frac{3}{8}$
3. $4\frac{4}{9} + 5\frac{2}{3}$
4. $1\frac{2}{5} + 3\frac{5}{6}$
5. $4\frac{1}{2} + 5\frac{3}{8}$
6. $5\frac{4}{7} + 3\frac{2}{3}$

Answers

1. $8\frac{11}{12}$
2. $13\frac{25}{24} = 14\frac{1}{24}$
3. $9\frac{10}{9} = 10\frac{1}{9}$
4. $4\frac{37}{30} = 5\frac{7}{30}$
5. $9\frac{7}{8}$
6. $8\frac{26}{21} = 9\frac{5}{21}$

MULTIPLICATION OF FRACTIONS

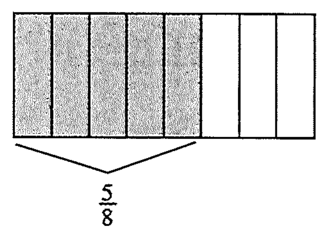
MULTIPLYING FRACTIONS WITH AN AREA MODEL

Multiplication of fractions is reviewed using a rectangular area model. Lines that divide the rectangle to represent one fraction are drawn vertically, and the correct number of parts are shaded. Then lines that divide the rectangle to represent the second fraction are drawn horizontally and part of the shaded region is darkened to represent the product of the two fractions.

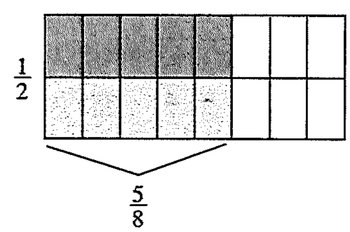
Example 1

$\frac{1}{2} \cdot \frac{5}{8}$ (that is, $\frac{1}{2}$ of $\frac{5}{8}$)

Step 1: Draw a generic rectangle and divide it into 8 pieces vertically. Lightly shade 5 of those pieces. Label it $\frac{5}{8}$.



Step 2: Use a horizontal line and divide the generic rectangle in half. Darkly shade $\frac{1}{2}$ of $\frac{5}{8}$ and label it.



Step 3: Write a number sentence. $\frac{1}{2} \cdot \frac{5}{8} = \frac{5}{16}$

The rule for multiplying fractions derived from the models above is to multiply the numerators, then multiply the denominators. Simplify the product when possible.

For additional information, see the Math Notes boxes in Lesson 5.1.4 of the *Core Connections, Course 1* text or Lesson 2.2.5 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 7A materials.

Example 2

a. $\frac{2}{3} \cdot \frac{2}{7} \Rightarrow \frac{2 \cdot 2}{3 \cdot 7} \Rightarrow \frac{4}{21}$

b. $\frac{3}{4} \cdot \frac{6}{7} \Rightarrow \frac{3 \cdot 6}{4 \cdot 7} \Rightarrow \frac{18}{28} \Rightarrow \frac{9}{14}$

Problems

Draw an area model for each of the following multiplication problems and write the answer.

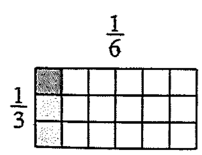
1. $\frac{1}{3} \cdot \frac{1}{6}$
2. $\frac{1}{4} \cdot \frac{3}{5}$
3. $\frac{2}{3} \cdot \frac{5}{9}$

Use the rule for multiplying fractions to find the answer for the following problems. Simplify when possible.

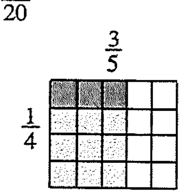
4. $\frac{1}{3} \cdot \frac{2}{5}$
5. $\frac{2}{3} \cdot \frac{2}{7}$
6. $\frac{3}{4} \cdot \frac{1}{5}$
7. $\frac{2}{5} \cdot \frac{2}{3}$
8. $\frac{2}{3} \cdot \frac{1}{4}$
9. $\frac{5}{6} \cdot \frac{2}{3}$
10. $\frac{4}{5} \cdot \frac{3}{4}$
11. $\frac{2}{15} \cdot \frac{1}{2}$
12. $\frac{3}{7} \cdot \frac{1}{2}$
13. $\frac{3}{8} \cdot \frac{4}{5}$
14. $\frac{2}{9} \cdot \frac{3}{5}$
15. $\frac{3}{10} \cdot \frac{5}{7}$
16. $\frac{5}{11} \cdot \frac{6}{7}$
17. $\frac{5}{6} \cdot \frac{3}{10}$
18. $\frac{10}{11} \cdot \frac{3}{5}$
19. $\frac{5}{12} \cdot \frac{3}{5}$
20. $\frac{7}{9} \cdot \frac{5}{14}$

Answers

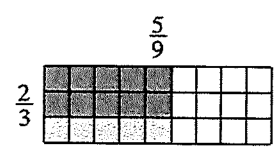
1. $\frac{1}{18}$



2. $\frac{3}{20}$



3. $\frac{10}{27}$



4. $\frac{2}{15}$
5. $\frac{4}{21}$
6. $\frac{3}{20}$
7. $\frac{4}{15}$
8. $\frac{2}{12} = \frac{1}{6}$
9. $\frac{10}{18} = \frac{5}{9}$
10. $\frac{12}{20} = \frac{3}{5}$
11. $\frac{2}{30} = \frac{1}{5}$
12. $\frac{3}{14}$
13. $\frac{12}{40} = \frac{3}{10}$
14. $\frac{6}{45} = \frac{2}{15}$
15. $\frac{15}{70} = \frac{3}{14}$
16. $\frac{30}{77}$
17. $\frac{15}{60} = \frac{1}{4}$
18. $\frac{30}{55} = \frac{6}{11}$
19. $\frac{15}{60} = \frac{1}{4}$
20. $\frac{35}{126} = \frac{5}{18}$

Example 2

a. $\frac{2}{3} \cdot \frac{2}{7} \Rightarrow \frac{2 \cdot 2}{3 \cdot 7} \Rightarrow \frac{4}{21}$

b. $\frac{3}{4} \cdot \frac{6}{7} \Rightarrow \frac{3 \cdot 6}{4 \cdot 7} \Rightarrow \frac{18}{28} \Rightarrow \frac{9}{14}$

Problems

Draw an area model for each of the following multiplication problems and write the answer.

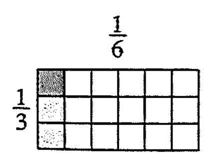
1. $\frac{1}{3} \cdot \frac{1}{6}$
2. $\frac{1}{4} \cdot \frac{3}{5}$
3. $\frac{2}{3} \cdot \frac{5}{9}$

Use the rule for multiplying fractions to find the answer for the following problems. Simplify when possible.

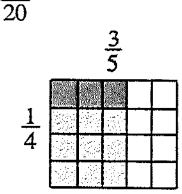
4. $\frac{1}{3} \cdot \frac{2}{5}$
5. $\frac{2}{3} \cdot \frac{2}{7}$
6. $\frac{3}{4} \cdot \frac{1}{5}$
7. $\frac{2}{5} \cdot \frac{2}{3}$
8. $\frac{2}{3} \cdot \frac{1}{4}$
9. $\frac{5}{6} \cdot \frac{2}{3}$
10. $\frac{4}{5} \cdot \frac{3}{4}$
11. $\frac{2}{15} \cdot \frac{1}{2}$
12. $\frac{3}{7} \cdot \frac{1}{2}$
13. $\frac{3}{8} \cdot \frac{4}{5}$
14. $\frac{2}{9} \cdot \frac{3}{5}$
15. $\frac{3}{10} \cdot \frac{5}{7}$
16. $\frac{5}{11} \cdot \frac{6}{7}$
17. $\frac{5}{6} \cdot \frac{3}{10}$
18. $\frac{10}{11} \cdot \frac{3}{5}$
19. $\frac{5}{12} \cdot \frac{3}{5}$
20. $\frac{7}{9} \cdot \frac{5}{14}$

Answers

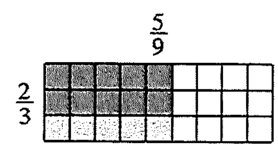
1. $\frac{1}{18}$



2. $\frac{3}{20}$



3. $\frac{10}{27}$



4. $\frac{2}{15}$
5. $\frac{4}{21}$
6. $\frac{3}{20}$
7. $\frac{4}{15}$
8. $\frac{2}{12} = \frac{1}{6}$
9. $\frac{10}{18} = \frac{5}{9}$
10. $\frac{12}{20} = \frac{3}{5}$
11. $\frac{2}{30} = \frac{1}{5}$
12. $\frac{3}{14}$
13. $\frac{12}{40} = \frac{3}{10}$
14. $\frac{6}{45} = \frac{2}{15}$
15. $\frac{15}{70} = \frac{3}{14}$
16. $\frac{30}{77}$
17. $\frac{15}{60} = \frac{1}{4}$
18. $\frac{30}{55} = \frac{6}{11}$
19. $\frac{15}{60} = \frac{1}{4}$
20. $\frac{35}{126} = \frac{5}{18}$

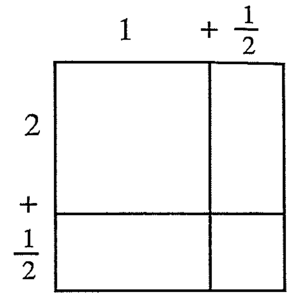
MULTIPLYING MIXED NUMBERS

There are two ways to multiply mixed numbers. One is with generic rectangles. For additional information, see the Math Notes box in Lesson 2.3.1 of the *Core Connections, Course 2* text.

Example 1

Find the product: $2\frac{1}{2} \cdot 1\frac{1}{2}$.

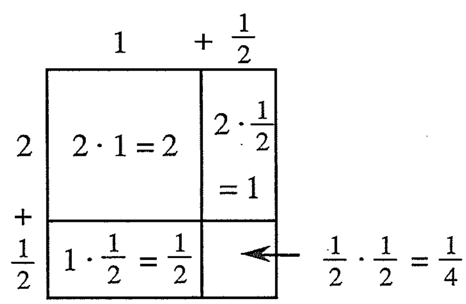
Step 1: Draw the generic rectangle. Label the top 1 plus $\frac{1}{2}$. Label the side 2 plus $\frac{1}{2}$.



Step 2: Write the area of each smaller rectangle in each of the four parts of the drawing.

Find the total area:

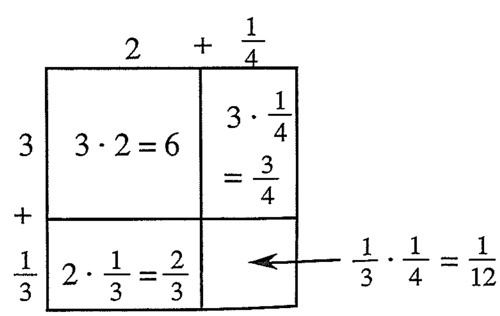
$2 + 1 + \frac{1}{2} + \frac{1}{4} = 3\frac{3}{4}$



Step 3: Write a number sentence: $2\frac{1}{2} \cdot 1\frac{1}{2} = 3\frac{3}{4}$

Example 2

Find the product: $3\frac{1}{3} \cdot 2\frac{1}{4}$.



$6 + \frac{3}{4} + \frac{2}{3} + \frac{1}{12} \Rightarrow 6 + \frac{9}{12} + \frac{8}{12} + \frac{1}{12} \Rightarrow$

$\Rightarrow 6\frac{18}{12} \Rightarrow 7\frac{1}{2}$

Problems

Use a generic rectangle to find each product.

1. $1\frac{1}{4} \cdot 1\frac{1}{2}$
2. $3\frac{1}{6} \cdot 2\frac{1}{2}$
3. $2\frac{1}{4} \cdot 1\frac{1}{2}$
4. $1\frac{1}{3} \cdot 1\frac{1}{6}$
5. $1\frac{1}{2} \cdot 1\frac{1}{3}$

Answers

1. $1\frac{7}{8}$

	1	$+\ \frac{1}{2}$
1	1	$\frac{1}{2}$
$+\ \frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$

2. $7\frac{11}{12}$

	2	$+\ \frac{1}{2}$
3	6	$\frac{3}{2}$
$+\ \frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{12}$

3. $3\frac{3}{8}$

	1	$+\ \frac{1}{2}$
2	2	1
$+\ \frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$

4. $1\frac{5}{9}$

	1	$+\ \frac{1}{6}$
1	1	$\frac{1}{6}$
$+\ \frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{18}$

5. 2

	1	$+\ \frac{1}{3}$
1	1	$\frac{1}{3}$
$+\ \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{6}$

You can also multiply mixed numbers by changing them to fractions greater than 1, then multiplying the numerators and multiplying the denominators. Simplify if possible.

For more information, see the Math Notes box in Lesson 5.2.1 of the *Core Connections, Course 1* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 7A materials.

Example 3

$2\frac{1}{2} \cdot 1\frac{1}{4} \Rightarrow \frac{5}{2} \cdot \frac{5}{4} \Rightarrow \frac{5 \cdot 5}{2 \cdot 4} \Rightarrow \frac{25}{8} \Rightarrow 3\frac{1}{8}$

Problems

Find each product, using the method of your choice. Simplify when possible.

1. $2\frac{1}{4} \cdot 1\frac{3}{8}$
2. $3\frac{3}{5} \cdot 2\frac{4}{7}$
3. $2\frac{3}{8} \cdot 1\frac{1}{6}$
4. $3\frac{7}{9} \cdot 2\frac{5}{8}$
5. $1\frac{2}{9} \cdot 2\frac{3}{7}$
6. $3\frac{4}{7} \cdot 5\frac{8}{11}$
7. $2\frac{3}{8} \cdot 1\frac{1}{16}$
8. $2\frac{8}{9} \cdot 2\frac{5}{8}$
9. $1\frac{1}{3} \cdot 1\frac{4}{7}$
10. $2\frac{1}{7} \cdot 2\frac{7}{10}$

Answers

1. $3\frac{3}{32}$
2. $9\frac{9}{35}$
3. $2\frac{37}{48}$
4. $9\frac{11}{12}$
5. $2\frac{61}{63}$
6. $20\frac{5}{11}$
7. $2\frac{67}{128}$
8. $7\frac{7}{12}$
9. $2\frac{2}{21}$
10. $5\frac{11}{14}$

DIVISION BY FRACTIONS

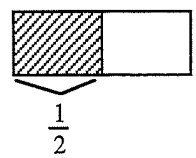
Division by fractions introduces three methods to help students understand how dividing by fractions works. In general, think of division for a problem like 8 ÷ 2 as, "In 8, how many groups of 2 are there?" Similarly, $\frac{1}{2} \div \frac{1}{4}$ means, "In $\frac{1}{2}$, how many fourths are there?"

For more information, see the Math Notes boxes in Lessons 7.2.2 and 7.2.4 of the *Core Connections, Course 1* text or Lesson 3.3.1 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 8B materials. The first two examples show how to divide fractions using a diagram.

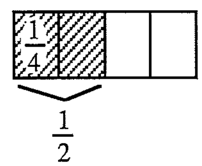
Example 1

Use the rectangular model to divide: $\frac{1}{2} \div \frac{1}{4}$.

Step 1: Using the rectangle, we first divide it into 2 equal pieces. Each piece represents $\frac{1}{2}$. Shade $\frac{1}{2}$ of it.



Step 2: Then divide the *original* rectangle into four equal pieces. Each section represents $\frac{1}{4}$. In the shaded section, $\frac{1}{2}$, there are 2 fourths.



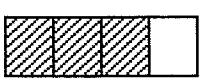
Step 3: Write the equation.

$\frac{1}{2} \div \frac{1}{4} = 2$

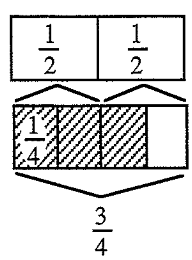
Example 2

In $\frac{3}{4}$, how many $\frac{1}{2}$'s are there?

That is, $\frac{3}{4} \div \frac{1}{2} = ?$



Start with $\frac{3}{4}$.



In $\frac{3}{4}$ there is one full $\frac{1}{2}$ shaded and half of another one (that is half of one-half).

So: $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$
(one and one-half halves)

Problems

Use the rectangular model to divide.

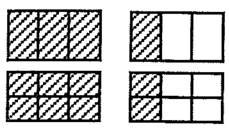
1. $1\frac{1}{3} \div \frac{1}{6}$ 2. $\frac{3}{2} \div \frac{3}{4}$ 3. $1 \div \frac{1}{4}$ 4. $1\frac{1}{4} \div \frac{1}{2}$ 5. $2\frac{2}{3} \div \frac{1}{9}$

Answers

1. 8

3rds

6ths

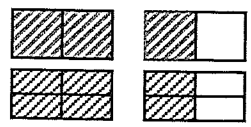


8 sixths

2. 2

halves

quarters

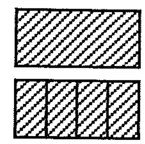


2 three fourths

3. 4

one

fourths

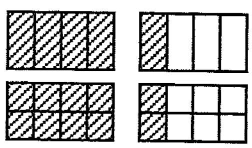


4 fourths

4. $2\frac{1}{2}$

quarters

halves

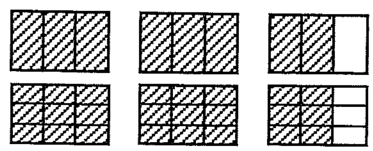


$2\frac{1}{2}$ halves

5. 24

3rds

9ths



24 ninths

The next two examples use common denominators to divide by a fraction. Express both fractions with a common denominator, then divide the first numerator by the second.

Example 3

$\frac{4}{5} \div \frac{2}{3} \Rightarrow \frac{12}{15} \div \frac{10}{15} \Rightarrow \frac{12}{10} \Rightarrow \frac{6}{5}$ or $1\frac{1}{5}$

Example 4

$1\frac{1}{3} \div \frac{1}{6} \Rightarrow \frac{4}{3} \div \frac{1}{6} \Rightarrow \frac{8}{6} \div \frac{1}{6} \Rightarrow \frac{8}{1}$ or 8

One more way to divide fractions is to use the Giant One from previous work with fractions to create a "Super Giant One." To use a Super Giant One, write the division problem in fraction form, with a fraction in both the numerator and the denominator. Use the reciprocal of the denominator for the numerator and the denominator in the Super Giant One, multiply the fractions as usual, and simplify the resulting fraction when possible.

Example 5

$$\frac{\frac{1}{2}}{\frac{1}{4}} \cdot \frac{\frac{4}{1}}{\frac{4}{1}} = \frac{\frac{4}{2}}{1} = \frac{4}{2} = 2$$

Example 6

$$\frac{\frac{3}{4}}{\frac{1}{6}} \cdot \frac{\frac{6}{1}}{\frac{6}{1}} = \frac{\frac{18}{4}}{1} = \frac{9}{2} = 4\frac{1}{2}$$

Example 7

$$\frac{1\frac{1}{3}}{1\frac{1}{2}} = \frac{\frac{4}{3}}{\frac{3}{2}} \cdot \frac{\frac{2}{3}}{\frac{2}{3}} = \frac{\frac{8}{9}}{1} = \frac{8}{9}$$

Example 8

$$\frac{2}{3} \div \frac{3}{5} \Rightarrow \frac{10}{15} \div \frac{9}{15} \Rightarrow \frac{10}{9}$$

Compared to:

$$\frac{\frac{2}{3}}{\frac{3}{5}} \cdot \frac{\frac{5}{3}}{\frac{5}{3}} = \frac{\frac{10}{9}}{1} = \frac{10}{9} = 1\frac{1}{9}$$

Problems

Complete the division problems below. Use any method.

1. $\frac{3}{7} \div \frac{1}{8}$
2. $1\frac{3}{7} \div \frac{1}{2}$
3. $\frac{4}{7} \div \frac{1}{3}$
4. $1\frac{4}{7} \div \frac{1}{3}$
5. $\frac{6}{7} \div \frac{5}{8}$
6. $\frac{3}{10} \div \frac{5}{7}$
7. $2\frac{1}{3} \div \frac{5}{8}$
8. $7 \div \frac{1}{3}$
9. $1\frac{1}{3} \div \frac{2}{5}$
10. $2\frac{2}{3} \div \frac{3}{4}$
11. $3\frac{1}{3} \div \frac{5}{6}$
12. $1\frac{1}{2} \div \frac{1}{2}$
13. $\frac{5}{8} \div 1\frac{1}{4}$
14. $10\frac{1}{3} \div \frac{1}{6}$
15. $\frac{3}{5} \div 6$

Answers

1. $3\frac{3}{7}$
2. $2\frac{6}{7}$
3. $1\frac{5}{7}$
4. $4\frac{5}{7}$
5. $1\frac{13}{35}$
6. $\frac{21}{50}$
7. $3\frac{11}{15}$
8. 21
9. $3\frac{1}{3}$
10. $3\frac{5}{9}$
11. 4
12. 3
13. $\frac{1}{2}$
14. 62
15. $\frac{1}{10}$

ARITHMETIC OPERATIONS WITH DECIMALS

ADDING AND SUBTRACTING DECIMALS: Write the problem in column form with the decimal points in a vertical column. Write in zeros so that all decimal parts of the number have the same number of digits. Add or subtract as with whole numbers. Place the decimal point in the answer aligned with those above.

MULTIPLYING DECIMALS: Multiply as with whole numbers. In the product, the number of decimal places is equal to the total number of decimal places in the factors (numbers you multiplied). Sometimes zeros need to be added to place the decimal point.

DIVIDING DECIMALS: When dividing a decimal by a whole number, place the decimal point in the answer space directly above the decimal point in the number being divided. Divide as with whole numbers. Sometimes it is necessary to add zeros to the number being divided to complete the division.

When dividing decimals or whole numbers by a decimal, the divisor must be multiplied by a power of ten to make it a whole number. The dividend must be multiplied by the same power of ten. Then divide following the same rules for division by a whole number.

For additional information, see the Math Notes boxes in Lesson 5.2.2 of the *Core Connections, Course 1* text, or Lessons 3.3.2 and 3.3.3 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 2, Checkpoint 7A, and Checkpoint 8B materials.

Example 1

Add 47.37, 28.9, 14.56, and 7.8.

47.37
28.9**0**
14.56
+ 7.8**0**
―――
98.63

Example 2

Subtract 198.76 from 473.2.

473.2**0**
– 198.76
―――
274.44

Example 3

Multiply 27.32 by 14.53.

27.32 (2 decimal places)
x 14.53 (2 decimal places)
―――
8196
13660
10928
2732
―――
396.9596 (4 decimal places)

Example 4

Multiply 0.37 by 0.0004.

0.37 (2 decimal places)
x 0.0004 (4 decimal places)
―――
0.000148 (6 decimal places)

Example 5

Divide 32.4 by 8.

$$8\overline{)32.40} = 4.05$$

32.4**0** ÷ 8:
32
―――
0 40
40
―――
0

Example 6

Divide 27.42 by 1.2. First multiply each number by 10^1 or 10.

$$1.2\overline{)27.42} \Rightarrow 12\overline{)274.2} \Rightarrow 12\overline{)274.20} = 22.85$$

24
―
34
24
―
102
96
―
60
60
―
0

ARITHMETIC OPERATIONS WITH DECIMALS

ADDING AND SUBTRACTING DECIMALS: Write the problem in column form with the decimal points in a vertical column. Write in zeros so that all decimal parts of the number have the same number of digits. Add or subtract as with whole numbers. Place the decimal point in the answer aligned with those above.

MULTIPLYING DECIMALS: Multiply as with whole numbers. In the product, the number of decimal places is equal to the total number of decimal places in the factors (numbers you multiplied). Sometimes zeros need to be added to place the decimal point.

DIVIDING DECIMALS: When dividing a decimal by a whole number, place the decimal point in the answer space directly above the decimal point in the number being divided. Divide as with whole numbers. Sometimes it is necessary to add zeros to the number being divided to complete the division.

When dividing decimals or whole numbers by a decimal, the divisor must be multiplied by a power of ten to make it a whole number. The dividend must be multiplied by the same power of ten. Then divide following the same rules for division by a whole number.

For additional information, see the Math Notes boxes in Lesson 5.2.2 of the *Core Connections, Course 1* text, or Lessons 3.3.2 and 3.3.3 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 2, Checkpoint 7A, and Checkpoint 8B materials.

Example 1

Add 47.37, 28.9, 14.56, and 7.8.

```
  47.37
  28.90
  14.56
 + 7.80
 ------
  98.63
```

Example 2

Subtract 198.76 from 473.2.

```
  473.20
- 198.76
--------
  274.44
```

Example 3

Multiply 27.32 by 14.53.

```
    27.32   (2 decimal places)
  x 14.53   (2 decimal places)
  -------
     8196
   13660
  10928
  2732
 --------
 396.9596   (4 decimal places)
```

Example 4

Multiply 0.37 by 0.0004.

```
   0.37     (2 decimal places)
 x 0.0004   (4 decimal places)
 --------
 0.000148   (6 decimal places)
```

Example 5

Divide 32.4 by 8.

```
   4.05
8)32.40
  32
  ----
   0 40
     40
   ----
      0
```

Example 6

Divide 27.42 by 1.2. First multiply each number by 10^1 or 10.

$1.2\overline{)27.42} \Rightarrow 12\overline{)274.2} \Rightarrow 12\overline{)274.20}$

```
      22.85
12)274.20
   24
   --
    34
    24
    --
    102
     96
    ---
      60
      60
      --
       0
```

Problems

1. $4.7 + 7.9$	2. $3.93 + 2.82$	3. $38.72 + 6.7$
4. $58.3 + 72.84$	5. $4.73 + 692$	6. $428 + 7.392$
7. $42.1083 + 14.73$	8. $9.87 + 87.47936$	9. $9.999 + 0.001$
10. $0.0001 + 99.9999$	11. $0.0137 + 1.78$	12. $2.037 + 0.09387$
13. $15.3 + 72.894$	14. $47.9 + 68.073$	15. $289.307 + 15.938$
16. $476.384 + 27.847$	17. $15.38 + 27.4 + 9.076$	18. $48.32 + 284.3 + 4.638$
19. $278.63 + 47.0432 + 21.6$	20. $347.68 + 28.00476 + 84.3$	21. $8.73 - 4.6$
22. $9.38 - 7.5$	23. $8.312 - 6.98$	24. $7.045 - 3.76$
25. $6.304 - 3.68$	26. $8.021 - 4.37$	27. $14 - 7.431$
28. $23 - 15.37$	29. $10 - 4.652$	30. $18 - 9.043$
31. $0.832 - 0.47$	32. $0.647 - 0.39$	33. $1.34 - 0.0538$
34. $2.07 - 0.523$	35. $4.2 - 1.764$	36. $3.8 - 2.406$
37. $38.42 - 32.605$	38. $47.13 - 42.703$	39. $15.368 + 14.4 - 18.5376$
40. $87.43 - 15.687 - 28.0363$	41. $7.34 \cdot 6.4$	42. $3.71 \cdot 4.03$
43. $0.08 \cdot 4.7$	44. $0.04 \cdot 3.75$	45. $41.6 \cdot 0.302$
46. $9.4 \cdot 0.0053$	47. $3.07 \cdot 5.4$	48. $4.023 \cdot 3.02$
49. $0.004 \cdot 0.005$	50. $0.007 \cdot 0.0004$	51. $0.235 \cdot 0.43$
52. $4.32 \cdot 0.0072$	53. $0.0006 \cdot 0.00013$	54. $0.0005 \cdot 0.00026$
55. $8.38 \cdot 0.0001$	56. $47.63 \cdot 0.000001$	57. $0.078 \cdot 3.1$
58. $0.043 \cdot 4.2$	59. $350 \cdot 0.004$	60. $421 \cdot 0.00005$

Divide. Round answers to the hundredth, if necessary.

61. 14.3 ÷ 8
62. 18.32 ÷ 5
63. 147.3 ÷ 6
64. 46.36 ÷ 12
65. 100.32 ÷ 24
66. 132.7 ÷ 28
67. 47.3 ÷ 0.002
68. 53.6 ÷ 0.004
69. 500 ÷ 0.004
70. 420 ÷ 0.05
71. 1.32 ÷ 0.032
72. 3.486 ÷ 0.012
73. 46.3 ÷ 0.011
74. 53.7 ÷ 0.023
75. 25.46 ÷ 5.05
76. 26.35 ÷ 2.2
77. 6.042 ÷ 0.006
78. 7.035 ÷ 0.005
79. 207.3 ÷ 4.4
80. 306.4 ÷ 3.2

Answers

1. 12.6
2. 6.75
3. 45.42
4. 131.14
5. 696.73
6. 435.392
7. 56.8383
8. 97.34936
9. 10.000
10. 100.0000
11. 1.7937
12. 2.13087
13. 88.194
14. 115.973
15. 305.245
16. 504.231
17. 51.856
18. 337.258
19. 347.2732
20. 459.98476
21. 4.13
22. 1.88
23. 1.332
24. 3.285
25. 2.624
26. 3.651
27. 6.569
28. 7.63
29. 5.348
30. 8.957
31. 0.362
32. 0.257
33. 1.2862
34. 1.547
35. 2.436
36. 1.394
37. 5.815
38. 4.427
39. 11.2304
40. 43.7067
41. 46.976
42. 14.9513
43. 0.376
44. 0.15
45. 12.5632
46. 0.04982
47. 16.578
48. 12.14946
49. 0.000020
50. 0.0000028
51. 0.10105
52. 0.031104
53. 0.000000078
54. 0.000000130
55. 0.000838
56. 0.0004763
57. 0.2418
58. 0.1806
59. 1.4
60. 0.02105
61. 1.7875 or 1.79
62. 3.664 or 3.66
63. 24.55
64. $3.86\overline{3}$ or 3.86
65. 4.18
66. 4.74
67. 23,650
68. 13,400
69. 125,000
70. 8400
71. 41.25
72. 29.05
73. 4209.09
74. 2334.78
75. 5.04
76. 11.98
77. 1007
78. 1407
79. 47.11
80. 95.75

MULTIPLYING DECIMALS AND PERCENTS

Understanding how many places to move the decimal point in a decimal multiplication problem is connected to the multiplication of fractions and place value.

Computations involving calculating "a percent of a number" are simplified by changing the percent to a decimal.

Example 1

Multiply $(0.2)\cdot(0.3)$.

In fractions this means $\frac{2}{10}\cdot\frac{3}{10}\Rightarrow\frac{6}{100}$.

Knowing that the answer must be in the hundredths place tells you how many places to move the decimal point (to the left) without using the fractions.

(tenths)(tenths) = hundredths

Therefore move two places.

$$\begin{array}{r} 0.2 \\ \underline{\times\ 0.3} \\ 0.06 \end{array}$$

Example 2

Multiply $(1.7)\cdot(0.03)$.

In fractions this means $\frac{17}{10}\cdot\frac{3}{100}\Rightarrow\frac{51}{1000}$.

Knowing that the answer must be in the thousandths place tells you how many places to move the decimal point (to the left) without using the fractions.

(tenths)(hundredths) = thousandths

Therefore move three places.

$$\begin{array}{r} 1.7 \\ \underline{\times\ 0.03} \\ 0.051 \end{array}$$

Example 3

Calculate 17% of 32.5 without using a calculator.

Since $17\% = \frac{17}{100} = 0.17$,

17% of $32.5 \Rightarrow (0.17)\cdot(32.5)$

$\Rightarrow 5.525$

$$\begin{array}{r} 32.5 \\ \underline{\times\ 0.17} \\ 2275 \\ \underline{3250} \\ 5.525 \end{array}$$

Problems

Identify the number of places to the left to move the decimal point in the product. Do not compute the product.

1.	$(0.3)\cdot(0.5)$	2.	$(1.5)\cdot(0.12)$	3.	$(1.23)\cdot(2.6)$
4.	$(0.126)\cdot(3.4)$	5.	$17\cdot(32.016)$	6.	$(4.32)\cdot(3.1416)$

Compute without using a calculator.

7.	$(0.8)\cdot(0.03)$	8.	$(3.2)\cdot(0.3)$	9.	$(1.75)\cdot(0.09)$
10.	$(4.5)\cdot(3.2)$	11.	$(1.8)\cdot(0.032)$	12.	$(7.89)\cdot(6.3)$
13.	8% of 540	14.	70% of 478	15.	37% of 4.7
16.	17% of 96	17.	15% of 4.75	18.	130% of 42

Answers

1.	2	2.	3	3.	3
4.	4	5.	3	6.	6
7.	0.024	8.	0.96	9.	0.1575
10.	14.4	11.	0.0576	12.	49.707
13.	43.2	14.	334.6	15.	1.739
16.	16.32	17.	0.7125	18.	54.6

COMBINING LIKE TERMS

Algebraic expressions can also be simplified by combining (adding or subtracting) terms that have the same variable(s) raised to the same powers, into one term. The skill of combining like terms is necessary for solving equations. For additional information, see the Math Notes box in Lesson 6.2.4 of the *Core Connections, Course 1* text, Lesson 4.3.2 of the *Core Connections, Course 2* text, or Lesson 2.1.3 of the *Core Connections, Course 3* text. For additional examples and practice, see the *Core Connections, Course 2* Checkpoint 7A materials.

Example 1

Combine like terms to simplify the expression $3x + 5x + 7x$.

All these terms have x as the variable, so they are combined into one term, $15x$.

Example 2

Simplify the expression $3x + 12 + 7x + 5$.

The terms with x can be combined. The terms without variables (the constants) can also be combined.

$3x + 12 + 7x + 5$

$3x + 7x + 12 + 5$

$10x + 17$

Note that in the simplified form the term with the variable is listed before the constant term.

Example 3

Simplify the expression $5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1$.

$5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1$

$4x^2 + 2x^2 + 5x + 2x + x + 10 - 6 - 1$

$6x^2 + 8x + 3$

Note that terms with the same variable but with different exponents are not combined and are listed in order of decreasing power of the variable, in simplified form, with the constant term last.

Example 4

The algebra tiles, as shown in the *Perimeter Using Algebra Tiles* section, are used to model how to combine like terms.

The large square represents x^2, the rectangle represents x, and the small square represents one. We can only combine tiles that are alike: large squares with large squares, rectangles with rectangles, and small squares with small squares. If we want to combine: $2x^2 + 3x + 4$ and $3x^2 + 5x + 7$, visualize the tiles to help combine the like terms:

$2x^2$ (2 large squares) + $3x$ (3 rectangles) + 4 (4 small squares)
+ $3x^2$ (3 large squares) + $5x$ (5 rectangles) + 7 (7 small squares)

The combination of the two sets of tiles, written algebraically, is: $5x^2 + 8x + 11$.

Example 5

Sometimes it is helpful to take an expression that is written horizontally, circle the terms with their signs, and rewrite *like* terms in vertical columns before you combine them:

$$(2x^2 - 5x + 6) + (3x^2 + 4x - 9)$$

$$\boxed{2x^2}\ \boxed{-5x}\ \boxed{+6} + \boxed{3x^2}\ \boxed{+4x}\ \boxed{-9}$$

$$\begin{array}{rrr} 2x^2 & -5x & +6 \\ +\ 3x^2 & +4x & -9 \\ \hline 5x^2 & -\ x & -3 \end{array}$$

This procedure may make it easier to identify the terms as well as the sign of each term.

Problems

Combine the following sets of terms.

1. $(2x^2 + 6x + 10) + (4x^2 + 2x + 3)$
2. $(3x^2 + x + 4) + (x^2 + 4x + 7)$
3. $(8x^2 + 3) + (4x^2 + 5x + 4)$
4. $(4x^2 + 6x + 5) - (3x^2 + 2x + 4)$
5. $(4x^2 - 7x + 3) + (2x^2 - 2x - 5)$
6. $(3x^2 - 7x) - (x^2 + 3x - 9)$
7. $(5x + 6) + (-5x^2 + 6x - 2)$
8. $2x^2 + 3x + x^2 + 4x - 3x^2 + 2$
9. $3c^2 + 4c + 7x - 12 + (-4c^2) + 9 - 6x$
10. $2a^2 + 3a^3 - 4a^2 + 6a + 12 - 4a + 2$

Answers

1. $6x^2 + 8x + 13$
2. $4x^2 + 5x + 11$
3. $12x^2 + 5x + 7$
4. $x^2 + 4x + 1$
5. $6x^2 - 9x - 2$
6. $2x^2 - 10x + 9$
7. $-5x^2 + 11x + 4$
8. $7x + 2$
9. $-c^2 + 4c + x - 3$
10. $3a^3 - 2a^2 + 2a + 14$

GRAPHING AND SOLVING INEQUALITIES

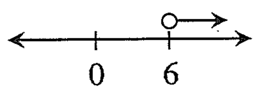
GRAPHING INEQUALITIES

The solutions to an equation can be represented as a point (or points) on the number line. If the expression comparison mat has a range of solutions, the solution is expressed as an inequality represented by a ray or segment with solid or open endpoints. Solid endpoints indicate that the endpoint is included in the solution ($\le$ or $\ge$), while the open dot indicates that it is not part of the solution ($<$ or $>$).

For additional information, see the Math Notes box in Lesson 7.3.4 of the *Core Connections, Course 1* text.

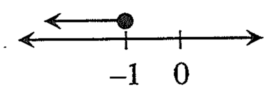
Example 1

$x > 6$



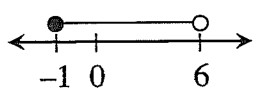
Example 2

$x \le -1$



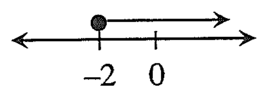
Example 3

$-1 \le y < 6$



Example 4

$y \ge -2$

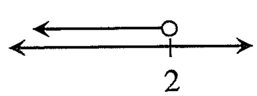


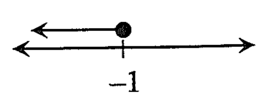
Problems

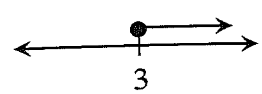
Graph each inequality on a number line.

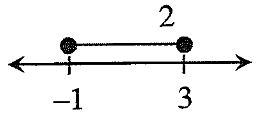
1. $m < 2$
2. $x \le -1$
3. $y \ge 3$
4. $-1 \le x \le 3$
5. $-6 < x < -2$
6. $-1 < x \le 2$
7. $m > -9$
8. $x \ne 1$
9. $x \le 3$

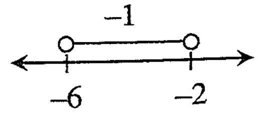
Answers

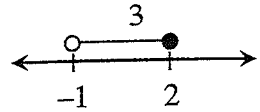
1. 

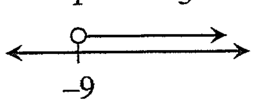
2. 

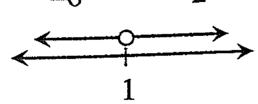
3. 

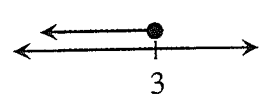
4. 

5. 

6. 

7. 

8. 

9. 

Problems

Solve each inequality.

1. $x+3>-1$
2. $y-3\le 5$
3. $-3x\le -6$
4. $2m+1\ge -7$
5. $-7<-2y+3$
6. $8\ge -2m+2$
7. $2x-1<-x+8$
8. $2(m+1)\ge m-3$
9. $3m+1\le m+7$

Answers

1. $x>-4$
2. $y\le 8$
3. $x\ge 2$
4. $m\ge -4$
5. $y<5$
6. $m\ge -3$
7. $x<3$
8. $m\ge -5$
9. $m\le 3$

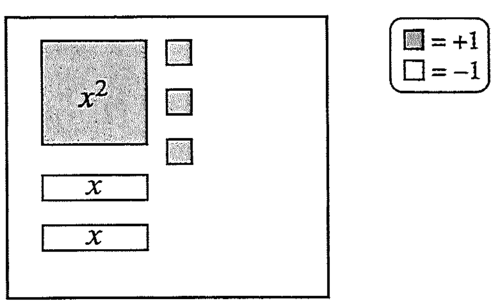
SIMPLIFYING EXPRESSIONS (ON AN EXPRESSION MAT)

Single Region Expression Mats (*Core Connections, Course 2*)

Algebra tiles and Expression Mats are concrete organizational tools used to represent algebraic expressions. Pairs of Expression Mats can be modified to make Expression Comparison Mats (see next section) and Equation Mats. Positive tiles are shaded and negative tiles are blank. A matching pair of tiles with one tile shaded and the other one blank represents zero (0).

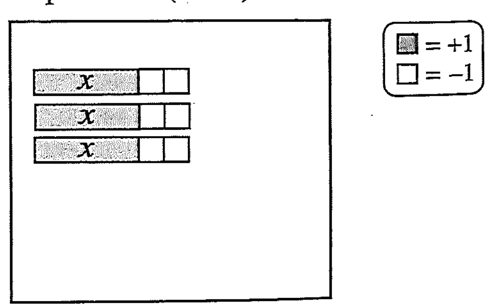
Example 1

Represent x^2-2x+3.



Example 2

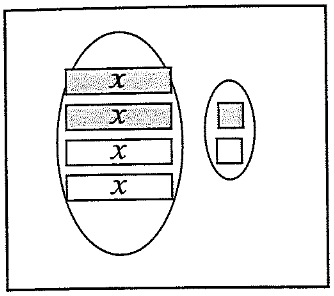
Represent $3(x-2)$.

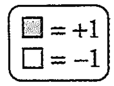


Note that $3(x-2)=3x-6$.

Example 3

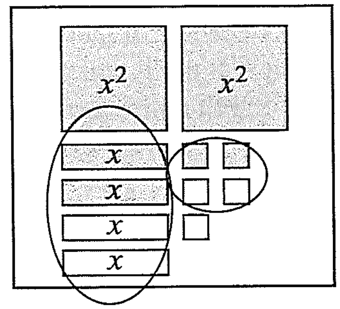
This expression makes zero.

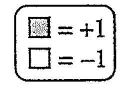




Example 4

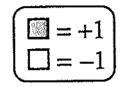
Simplify $2x^2 + 2x + 2 + (-2x) + (-3)$.



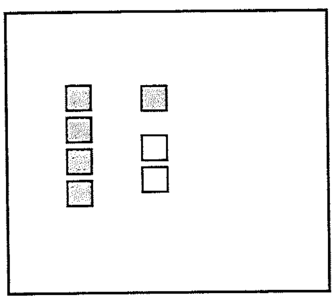


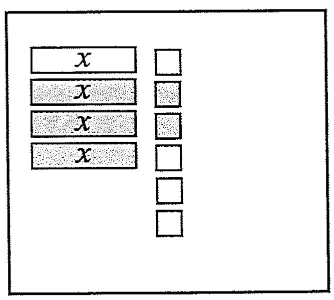
After removing zeros, $2x^2 - 1$ remains.

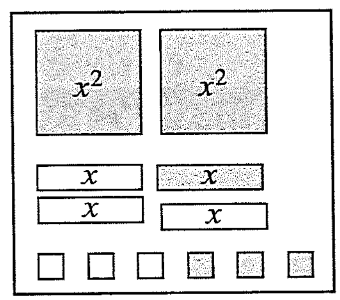
Problems

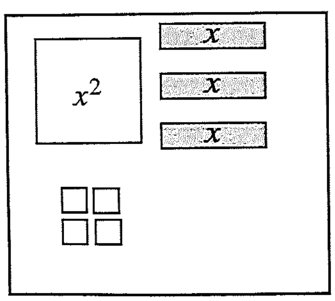


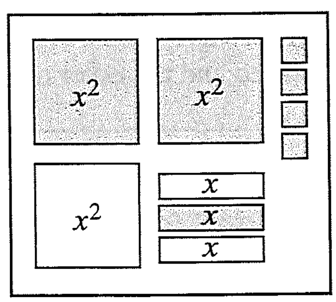
Simplify each expression.

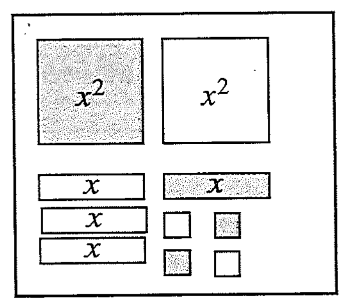
1. 

2. 

3. 

4. 

5. 

6. 

7. $2x - 3 + x + 1$
8. $-3x + 2x + 4$
9. $x^2 - 2x + 3 + 3x - 1$
10. $x + (-3) + 5 - 2x$
11. $-3 + 2x + (-1) - 4x$
12. $3(x + 3) - 2x$
13. $x^2 - 2x + 3 - 2x^2 + 1$
14. $2(x - 2) + 3 - x$
15. $2(x^2 + 3) + 2x - 1$

Answers

1.	3	2.	$2x-2$	3.	$2x^2-2x$
4.	$-x^2+3x-4$	5.	x^2-x+4	6.	$-2x$
7.	$3x-2$	8.	$-x+4$	9.	x^2+x+2
10.	$-x+2$	11.	$-2x-4$	12.	$x+9$
13.	$-x^2-2x+4$	14.	$x-1$	15.	$2x^2+2x+5$

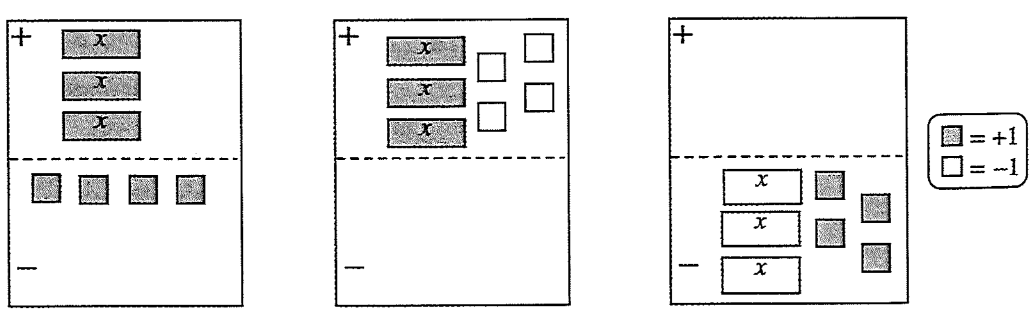
Two Region Expression Mats (*Core Connections, Course 3*)

An Expression Mat is an organizational tool that is used to represent algebraic expressions. Pairs of Expression Mats can be modified to make an Equation Mat. The upper half of an Expression Mat is the positive region and the lower half is the negative region. Positive algebra tiles are shaded and negative tiles are blank. A matching pair of tiles with one tile shaded and the other one blank represents zero (0).

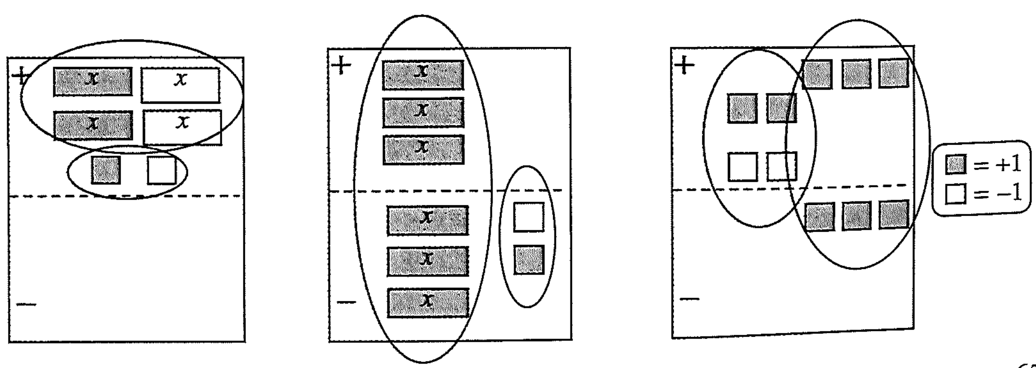
Tiles may be removed from or moved on an expression mat in one of three ways: (1) removing the same number of opposite tiles in the same region; (2) flipping a tile from one region to another. Such moves create "opposites" of the original tile, so a shaded tile becomes un-shaded and an un-shaded tile becomes shaded; and (3) removing an equal number of identical tiles from both the "+" and "-" regions. See the Math Notes box in Lesson 2.1.6 of the *Core Connections, Course 3* text.

Examples

$3x-4$ can be represented various ways.



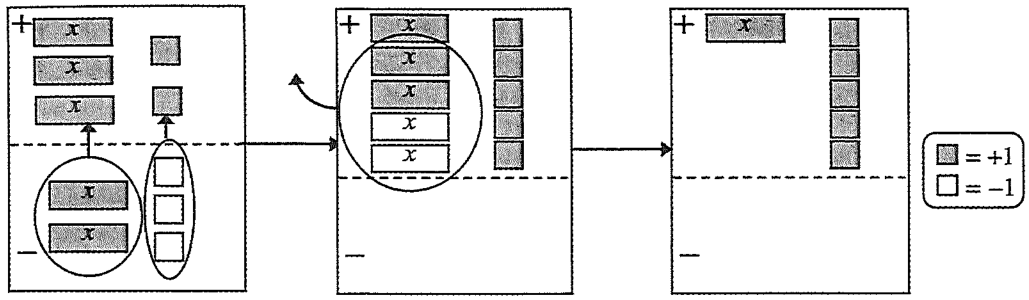
The Expression Mats at right all represent zero.



Example 1

$3x + 2 - (2x - 3)$

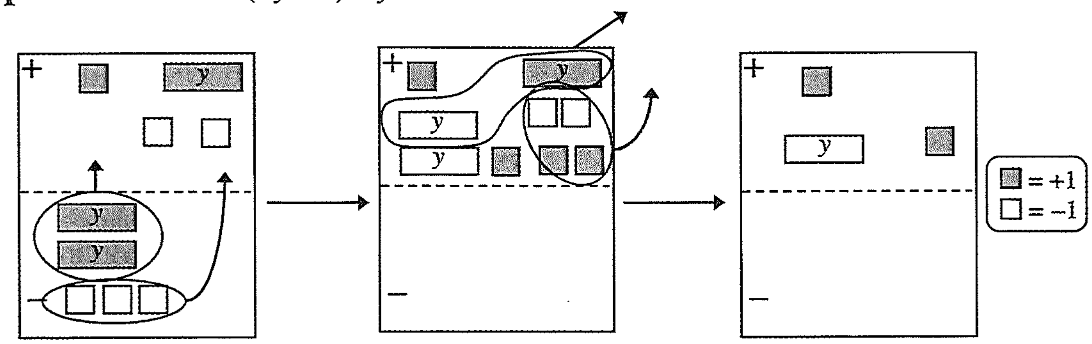
Expressions can be simplified by moving tiles to the top (change the sign) and looking for zeros.



$3x + 2 - (2x - 3)$ $\quad$ $3x + 2 - 2x + 3$ $\quad$ $x + 5$

Example 2

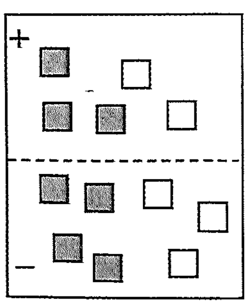
$1 - (2y - 3) + y - 2$

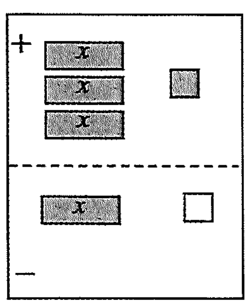


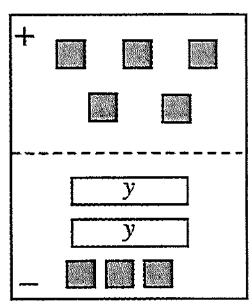
$1 - (2y - 3) + y - 2$ $\quad$ $1 - 2y + 3 + y - 2$ $\quad$ $-y + 2$

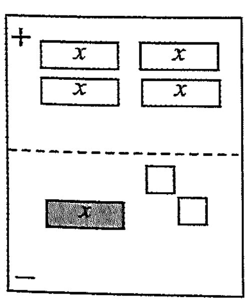
Problems

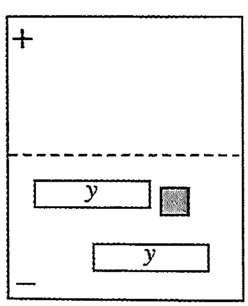
Simplify each expression.

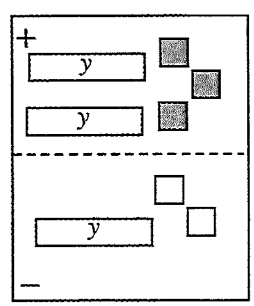
1. 

2. 

3. 

4. 

5. 

6. 

7.	$3+5x-4-7x$	8.	$-x-4x-7$	9.	$-(-x+3)$
10.	$4x-(x+2)$	11.	$5x-(-3x+2)$	12.	$x-5-(2-x)$
13.	$1-2y-2y$	14.	$-3x+5+5x-1$	15.	$3-(y+5)$
16.	$-(x+y)+4x+2y$	17.	$3x-7-(3x-7)$	18.	$-(x+2y+3)-3x+y$

Answers

1.	0	2.	$2x+2$	3.	$2y+2$
4.	$-5x+2$	5.	$2y-1$	6.	$-y+5$
7.	$-2x-1$	8.	$-5x-7$	9.	$x-3$
10.	$3x-2$	11.	$8x-2$	12.	$2x-7$
13.	$-4y+1$	14.	$2x+4$	15.	$-y-2$
16.	$3x+y$	17.	0	18.	$-4x-y-3$

COMPARING QUANTITIES (ON AN EXPRESSION MAT)

Combining two Expression Mats into an Expression Comparison Mat creates a concrete model for simplifying (and later solving) inequalities and equations.

Tiles may be removed or moved on the mat in the following ways:

(1) Removing the same number of opposite tiles (zeros) on the same side;

(2) Removing an equal number of identical tiles (balanced set) from both the left and right sides;

(3) Adding the same number of opposite tiles (zeros) on the same side; and

(4) Adding an equal number of identical tiles (balanced set) to both the left and right sides.

These strategies are called "legal moves."

After moving and simplifying the Expression Comparison Mat, students are asked to tell which side is greater. Sometimes it is only possible to tell which side is greater if you know possible values of the variable.

SOLVING EQUATIONS

Using a Two Region Equation Mat (*Core Connections, Course 2*)

Students combined two Expression Mats to figure out what value(s) of the variable make(s) one expression greater than the other. Now two Expression Mats are combined into an Equation Mat as a concrete model for solving equations. Practicing solving equations using this model will help students transition to solving equations abstractly with better accuracy and understanding.

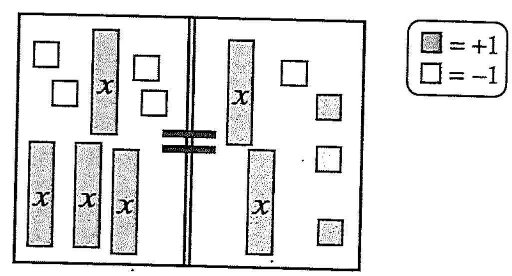
In general, and as shown in the example below, start by simplifying the Expression Mat. Next, isolate the variables on one side of the Equation Mat and the non-variables (unit tiles) on the other by adding/removing balanced sets and zeros. Then determine the value of the variable. Students are expected to be able to record and explain their steps.

For additional information, see the Math Notes box in Lesson 6.2.1 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 2* Checkpoint 8 materials.

Procedure and Example

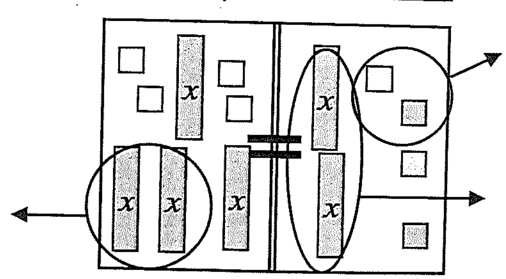
Solve: $x+(-4)+3x=2x-1+3$

First build the equation on the Equation Mat.



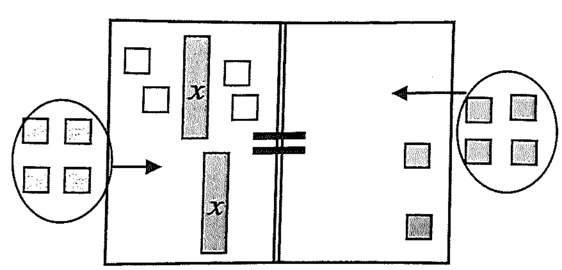
Second, simplify by removing zeros (−1 and +1 on the right side of the mat).

Third, remove a balanced set (2*x*) from both sides.



$4x-4=2x+2$

Isolate the variable by adding a balanced set (+4) to both sides and remove the zeros on the left side.

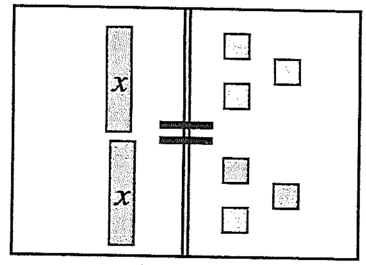


$2x-4=2$

Example continues on next page →

Example continued from previous page.

Finally, since both sides of the equation are equal, determine the value of x by dividing.



$2x = 6$

$x = 3$

Once the students understand how to solve equations using an Equation Mat, they may use the visual experience of moving the tiles to solve equations with variables and numbers. The procedures for moving variables and numbers in the solving process follow the same rules.

Note: When the process of solving an equation ends with different numbers on each side of the equal sign (for example $2 = 4$), there is *no solution* to the problem. When the result is the same expression or number on each side of the equation (for example, $x + 2 = x + 2$) it means that *all numbers* are solutions. For more information about these special cases, see the Math Note box in Lesson 6.2.6 of the *Core Connections, Course 2* text.

Example 1

Solve $3x + 3x - 1 = 4x + 9$

Solution		
	$3x + 3x - 1 = 4x + 9$	problem
	$6x - 1 = 4x + 9$	simplify by combining like terms
	$2x = 10$	add 1, subtract $4x$ on each side
	$x = 5$	divide by 2

Example 2

Solve $-2x + 1 + 3(x - 1) = -4 + -x - 2$

Solution		
	$-2x + 1 + 3(x - 1) = -4 + -x - 2$	problem
	$-2x + 1 + 3x - 3 = -x - 6$	distribute the 3 on the left side
	$x - 2 = -x - 6$	simplify by combining like terms
	$2x = -4$	add x, add 2 to each side
	$x = -2$	divide by 2

Problems

Solve each equation.

1. $3x+2+x=x+5$
2. $4x-2-2x=x-5$
3. $2x-3=-x+3$
4. $1+3x-x=x-4+2x$
5. $4-3x=2x-6$
6. $3+3x-x+2=3x+4$
7. $-x-3=2x-6$
8. $-4+3x-1=2x+1+2x$
9. $-x+3=6$
10. $5x-3+2x=x+2+x$
11. $2x-7=-x-1$
12. $-2+3x=x-2-4x$
13. $-3x+7=x-1$
14. $1+2x-4=-3+x$
15. $3(x+2)=x+2$
16. $2(x-2)+x=5$
17. $10=x+5+x$
18. $-x+2=x-5-3x$
19. $3(4+x)=x+6$
20. $6-x-3=4(x-2)$

Answers

1.	1	2.	–3	3.	2	4.	5	5.	2
6.	1	7.	1	8.	–6	9.	–3	10.	1
11.	2	12.	0	13.	2	14.	0	15.	–2
16.	3	17.	$2\frac{1}{2}$	18.	–7	19.	–3	20.	$2\frac{1}{5}$

Using a Four Region Equation Mat (*Core Connections, Course 3*)

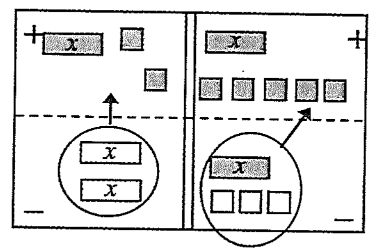
Combining two Expression Mats into an Equation Mat creates a concrete model for solving equations. Practice solving equations using the model will help students transition to solving equations abstractly with better accuracy and understanding.

In general, and as shown in the first example below, the negative in front of the parenthesis causes everything inside to "flip" from the top to the bottom or the bottom to the top of an Expression Mat, that is, all terms in the expression change signs. After simplifying the parentheses, simplify each Expression Mat. Next, isolate the variables on one side of the Equation Mat and the non-variables on the other side by removing matching tiles from both sides. Then determine the value of the variable. Students should be able to explain their steps. See the Math Notes boxes in Lessons 2.1.9 and 3.2.3 of the *Core Connections, Course 3* text. For additional examples and practice, see the *Core Connections, Course 3* Checkpoint 5 materials.

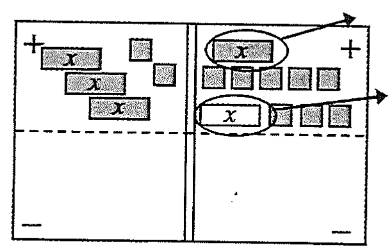
Procedure and Example

Solve $x+2-(-2x)=x+5-(x-3)$.

First build the equation on the Equation Mat.

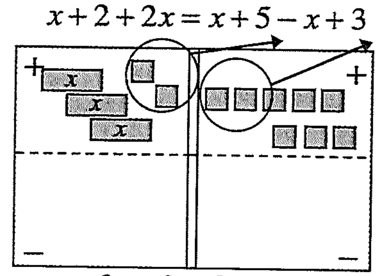


Second, simplify each side using legal moves on each Expression Mat, that is, on each side of the Equation Mat.



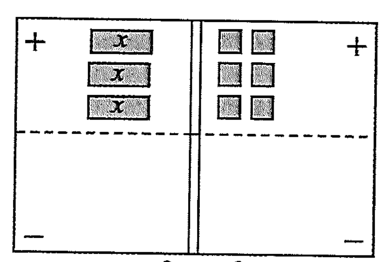
$x+2+2x=x+5-x+3$

Isolate x-terms on one side and non-x-terms on the other by removing matching tiles from both sides of the equation mat.



$3x+2=8$

Finally, since both sides of the equation are equal, determine the value of x.



$3x=6$

$x=2$

Once students understand how to solve equations using an Equation Mat, they may use the visual experience of moving tiles to solve equations with variables and numbers. The procedures for moving variables and numbers in the solving process follow the same rules.

Note: When the process of solving an equation ends with different numbers on each side of the equal sign (for example, $2 = 4$), there is *no solution* to the problem. When the result is the same expression or number on each side of the equation (for example, $x+2=x+2$) it means that *all numbers* are solutions. See the Math Notes box in Lesson 3.2.4 of the *Core Connections, Course 3* text.

Example 1 Solve $3x+3x-1=4x+9$

Solution		
	$3x+3x-1=4x+9$	problem
	$6x-1=4x+9$	simplify
	$2x=10$	add 1, subtract $4x$ on each side
	$x=5$	divide

Example 2 Solve $-2x+1-(-3x+3)=-4+(-x-2)$

Solution		
	$-2x+1-(-3x+3)=-4+(-x-2)$	problem
	$-2x+1+3x-3=-4-x-2$	remove parenthesis (flip)
	$x-2=-x-6$	simplify
	$2x=-4$	add x, add 2 to each side
	$x=-2$	divide

Problems

Solve each equation.

1. $2x-3=-x+3$
2. $1+3x-x=x-4+2x$
3. $4-3x=2x-6$
4. $3+3x-(x-2)=3x+4$
5. $-(x+3)=2x-6$
6. $-4+3x-1=2x+1+2x$
7. $-x+3=10$
8. $5x-3+2x=x+7+6x$
9. $4y-8-2y=4$
10. $9-(1-3y)=4+y-(3-y)$
11. $2x-7=-x-1$
12. $-2-3x=x-2-4x$
13. $-3x+7=x-1$
14. $1+2x-4=-3-(-x)$
15. $2x-1-1=x-3-(-5+x)$
16. $-4x-3=x-1-5x$
17. $10=x+6+2x$
18. $-(x-2)=x-5-3x$
19. $6-x-3=4x-8$
20. $0.5x-(-x+3)=x-5$

Answers

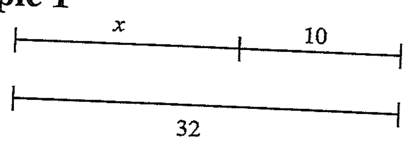
1. $x=2$	2. $x=5$	3. $x=2$	4. $x=1$	5. $x=1$
6. $x=-6$	7. $x=-7$	8. no solution	9. $x=6$	10. $x=-7$
11. $x=2$	12. all numbers	13. $x=2$	14. $x=0$	15. $x=2$
16. no solution	17. $x=1\frac{1}{3}$	18. $x=-7$	19. $x=2\frac{1}{5}$	20. $x=-4$

SOLVING EQUATIONS IN CONTEXT

Initially, equations are solved either by applying math facts (for example, $4x=12$, since $4\cdot 3=12$, $x=3$) or by matching equal quantities, simplifying the equation, and using math facts as shown in the examples below. Equations are often written in the context of a geometric situation.

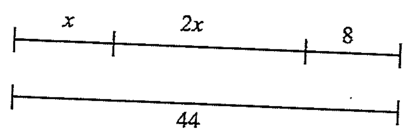
Write an equation that represents each situation and find the value of the variable.

Example 1



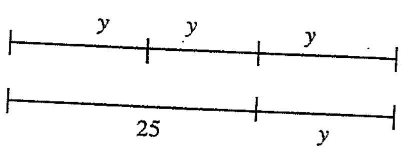
$$x+10=32$$
$$x=22$$

Example 2



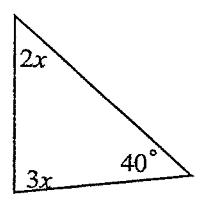
$$x+2x+8=44$$
$$x+2x=36$$
$$3x=36$$
$$x=12$$

Example 3



$$3y=25+y$$
$$2y=25$$
$$y=12.5$$

Example 4



$$2x+3x+40=180$$
$$2x+3x=140$$
$$5x=140$$
$$x=18$$